

# Charmed Baryon $\Lambda_c$ in Nuclear Matter

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Density dependences of the mass and self-energies of  $\Lambda_c$  in nuclear matter are studied in the parity projected QCD sum rule. Effects of nuclear matter are taken into account through the quark and gluon condensates. It is found that the four-quark condensates give dominant contributions. As the density dependences of the four-quark condensates are not known well, we examine two hypotheses. One is based on the factorization hypothesis (F-type) and the other is derived from the perturbative chiral quark model (QM-type). The F-type strongly depends on density, while the QM-type gives a weaker dependence. It is found that, for the F-type dependence, the energy of  $\Lambda_c$  increases as the density of nuclear matter grows, that is,  $\Lambda_c$  feels repulsion. On the other hand, the QM-type predicts a weak attraction ( $\sim 20$  MeV at the normal nuclear density) for  $\Lambda_c$  in nuclear matter. We carry out a similar analysis of the  $\Lambda$  hyperon and find that the F-type density dependence is too strong to explain the observed binding energy of  $\Lambda$  in nuclei. Thus we conclude that the weak density dependence of the four-quark condensate is more realistic. The scalar and vector self-energies of  $\Lambda_c$  for the QM-type dependence are found to be much smaller than those of the light baryons.

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## I. INTRODUCTION

Heavy hadrons attract much interest because they have new properties that are not seen in light hadrons. One of the interesting features of the heavy hadron physics is that the dynamics of light-quark part can be separated from the heavy quark due to the heavy quark spin symmetry [1]. The spin of heavy quark and that of the rest of the hadron become independent in the heavy quark mass limit. Therefore, one naively expects that the dynamics of light quarks in a heavy hadron can be analyzed more effectively. For instance, the heavy baryon consisting of a heavy quark and two light quarks can be used in a study of light di-quarks. Investigation of the heavy baryon in nuclear matter may help us to understand the relation between the partial restoration of chiral symmetry and the light di-quark.

The in-medium modification of the heavy baryon in nuclear matter is also interesting from the viewpoint of the heavy baryon-nucleon interaction. The N-N and Y-N interactions have been studied both theoretically and experimentally and understood fairly well. On the other hand, the properties of the interaction between the charmed (or bottomed) baryon and the nucleon are still open issue. The existence of  $\Lambda_c$  nuclei was discussed about 40 years ago [2] and subsequently studied in Refs. [3–10]. Recently, the  $\Lambda_c$ -N interaction were reinvestigated by using elaborated models [11, 12] and by lattice QCD simulation [13].

In this study, we investigate the in-medium properties of the  $\Lambda_c$  baryon from QCD sum rule. We additionally study the properties of the  $\Lambda$  hyperon and  $\Lambda_b$  and

compare their properties. The QCD sum rule analysis was developed by Shifman *et al.* [14, 15] and applied to the baryonic channel by Ioffe [16]. After that, the method was applied to the analyses in nuclear matter [17]. The  $\Lambda_Q$  baryon (Q denotes the heavy quark in general) was first investigated by Shuryak [18] in the heavy quark mass limit. Since then, the sum rule was continuously improved [19, 20] and the  $1/m_Q$  corrections [21] and  $\alpha_s$  corrections [22] were taken into account. QCD sum rules with finite heavy quark mass were also constructed in Ref. [23] and extended to the inclusion of the first order  $\alpha_s$  corrections [24–26] and parity projection [27]. Recently, the analyses are applied to the study of  $\Lambda_c$  in nuclear matter [28, 29]. However, their results are not consistent with each other. The result of Ref. [28] indicates that the energy of the  $\Lambda_c$  baryon increases and thus  $\Lambda_c$  feels a net repulsive potential. On the other hand, the result of Ref. [29] shows that there is large attractive interaction. For both cases, the  $\alpha_s$  corrections, which are pointed out as large contributions [24–26], are not included and parity projection is not done. We carry out a new analysis of the parity projected  $\Lambda_c$ ,  $\Lambda$  and  $\Lambda_b$  QCD sum rules including the  $\alpha_s$  corrections in nuclear matter with the Gaussian kernel. Advantages of using the Gaussian sum rule were discussed in Refs. [30–32].

The paper is organized as follows. In Sec. II, we introduce the  $\Lambda_c$  correlation function and construct the parity-projected in-medium QCD sum rules. The results of the analyses are summarized in Sec. III, where the density dependence of the energy (the pole position of the  $\Lambda_c$  propagator), the effective mass and the vector self-energy are presented. Next, we investigate effects of the density dependence of the four-quark condensate to the results in Sec. IV. The in-medium modification of  $\Lambda$  and  $\Lambda_b$  are also studied in the same section. Summary and conclusions are given in Sec. V.

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## II. $\Lambda_c$ CORRELATION FUNCTION

We consider the two point correlation function of  $\Lambda_c$  in nuclear matter:

$$\Pi^T(q) = i \int e^{iqx} d^4x \times \langle \Psi_0(\rho, u^\mu) | T[J_{\Lambda_c}(x) \bar{J}_{\Lambda_c}(0)] \Psi_0(\rho, u^\mu) \rangle, \quad (1)$$

where  $\Psi_0(\rho, u^\mu)$  is the ground state of nuclear matter, which is characterized by its velocity  $u^\mu$  and the nucleon density  $\rho$ .  $J_{\Lambda_c}$  is an interpolating operator which has the same quantum numbers of the  $\Lambda_c$  ground state. Without derivative, there are three independent interpolating operators [18]:

$$\begin{aligned} J_{\Lambda_c}^1(x) &= \epsilon^{abc} (u^{Ta}(x) C d^b(x)) \gamma_5 c^c(x), \\ J_{\Lambda_c}^2(x) &= \epsilon^{abc} (u^{Ta}(x) C \gamma_5 d^b(x)) c^c(x), \\ J_{\Lambda_c}^3(x) &= \epsilon^{abc} (u^{Ta}(x) C \gamma_5 \gamma_\mu d^b(x)) \gamma^\mu c^c(x). \end{aligned} \quad (2)$$

Here,  $a, b$  and  $c$  are color indices,  $C = i\gamma_0\gamma_2$  stands for the charge conjugation matrix and the spinor indices are omitted for simplicity. Each interpolating operator has a different type of di-quark, namely the  $J_{\Lambda_c}^1(x)$ ,  $J_{\Lambda_c}^2(x)$  and  $J_{\Lambda_c}^3(x)$  contain the component of the pseudo scalar, the scalar and the vector di-quarks, respectively. Based on the feature that the scalar di-quark  $\epsilon^{abc}(q^{Ta}C\gamma_5q^b)$  is the most attractive channel [33–37], we employ  $J_{\Lambda_c}^2$  as the  $\Lambda_c$  interpolating operator. In the following, we denote  $J_{\Lambda_c}^2$  as  $J_{\Lambda_c}$  for simplicity. With the help of the Lorentz covariance, parity invariance and time reversal invariance of the nuclear matter ground state, the correlation function of Eq. (1) can be decomposed into three scalar functions [38]:

$$\Pi^T(q) = \not{q} \Pi_1^T(q_0, |\vec{q}|) + \Pi_2^T(q_0, |\vec{q}|) + \not{u} \Pi_3^T(q_0, |\vec{q}|) \quad (3)$$

The variables of each scalar function,  $\Pi_i^T$  ( $i = 1, 2, 3$ ), are  $(q^2, q \cdot u)$ , but we write them as  $(q_0, |\vec{q}|)$  because we later take the rest frame of nuclear matter.

Generally, a baryon interpolating operator couples both to positive parity states and negative parity states because the positive parity interpolating operator  $J^+(x)$  is related to the negative parity interpolating operator:  $J^+(x) = \gamma_5 J^-(x)$  [39]. The extra  $\gamma_5$  only changes the sign of  $\Pi_2^T$  in Eq. (3). The method of the parity projection was proposed by Jido *et al.* [40] and Kondo *et al.* [41] and was successful in investigating the mass of the nucleon ground state and its negative parity excited state in vacuum. The parity projected QCD sum rule has been improved to include the  $\alpha_s$  corrections [42] and was applied to the analyses in nuclear matter [43].

The parity projected QCD sum rule can be constructed from the “forward-time” correlation function<sup>1</sup> [40, 42,

43]:

$$\begin{aligned} \Pi_m(q_0, |\vec{q}|) &= i \int d^4x e^{iqx} \theta(x_0) \\ &\quad \times \langle \Psi_0(\rho, u^\mu) | T[\eta(x) \bar{\eta}(0)] \Psi_0(\rho, u^\mu) \rangle \quad (4) \\ &= \not{q} \Pi_{m1}(q_0, |\vec{q}|) + \Pi_{m2}(q_0, |\vec{q}|) + \not{u} \Pi_{m3}(q_0, |\vec{q}|) \end{aligned}$$

The essential difference from the time-ordered correlation function is the insertion of the Heaviside step-function  $\theta(x_0)$  before carrying out the Fourier transform. This correlator contains contributions only from the states which propagate forward in time. Operating the parity projection operator  $P^\pm = \frac{\gamma_0 \pm 1}{2}$  to  $\Pi_m(q_0, |\vec{q}|)$  and taking the trace over the spinor index, the parity projected correlation functions can be derived:

$$\begin{aligned} \Pi_m^+(q_0, |\vec{q}|) &\equiv q_0 \Pi_{m1}(q_0, |\vec{q}|) + \Pi_{m2}(q_0, |\vec{q}|) \\ &\quad + u_0 \Pi_{m3}(q_0, |\vec{q}|) \\ \Pi_m^-(q_0, |\vec{q}|) &\equiv q_0 \Pi_{m1}(q_0, |\vec{q}|) - \Pi_{m2}(q_0, |\vec{q}|) \\ &\quad + u_0 \Pi_{m3}(q_0, |\vec{q}|). \end{aligned} \quad (5)$$

Note that the parity projection can be carried out in accordance with that in vacuum because it is based on the invariance of the ground state of nuclear matter under the parity transformation.

The QCD sum rule can be derived from the analyticity of the correlation function. The correlation functions  $\Pi_{m\text{OPE}}^\pm$  which are calculated by the operator product expansion (OPE) in deep Euclidean region ( $q^2 \rightarrow -\infty$ ) can be expressed by the hadronic spectral function  $\rho_m^\pm$  by the use of the dispersion relation:

$$\begin{aligned} \int_{-\infty}^{\infty} \text{Im}[\Pi_{m\text{OPE}}^\pm(q_0, |\vec{q}|)] W(q_0) dq_0 \\ = \pi \int_0^{\infty} \rho_m^\pm(q_0, |\vec{q}|) W(q_0) dq_0. \end{aligned} \quad (6)$$

Here, we have introduced a weighting function  $W(q_0)$ , which is real at real  $q_0$  and analytic in the upper half of the imaginary plane of  $q_0$ . The details of the derivation of Eq. (6) are explained in Ref. [42]. Using the above equation, we investigate the spectral function and the in-medium properties of  $\Lambda_c$ . The specific forms of  $\Pi_{m\text{OPE}}^\pm$  and  $\rho_m^\pm$  will be discussed in the next subsections.

### A. OPE of the $\Lambda_c$ correlation function

We first calculate the dimension 7 and 8 terms in the time-ordered correlation function and then construct the OPE of the “forward-time” correlation function including the first order  $\alpha_s$  corrections in nuclear matter taken from

<sup>1</sup> Note that in Ref. [40], this correlator was called the “old-fashioned” correlator.

the previous studies of Refs. [26, 28]:

$$\begin{aligned}
\Pi_{m\text{OPE}}(q_0, |\vec{q}|) &= i \int e^{iqx} dx \theta(x_0) \langle \Psi_0 | T \{ J_{\Lambda_c}(x) \bar{J}_{\Lambda_c}(0) \} | \Psi_0 \rangle \\
&= \not{q} \Pi_{m1\text{OPE}}(q_0, |\vec{q}|) + \Pi_{m2\text{OPE}}(q_0, |\vec{q}|) \\
&\quad + \not{q} \Pi_{m3\text{OPE}}(q_0, |\vec{q}|)
\end{aligned} \tag{7}$$

Explicit expressions of  $\rho_{mi\text{OPE}} \equiv \frac{1}{\pi} \text{Im}[\Pi_{mi\text{OPE}}]$  ( $i = 1, 2, 3$ ) are given as

$$\begin{aligned}
q_0 \rho_{m1\text{OPE}}(q_0, |\vec{q}|)|_{\vec{q}=0} &= q_0 \rho_{m1\text{OPE}}^{\text{pert}}(q_0) + q_0 \rho_{m1\text{OPE}}^{\text{cond}}(q_0),
\end{aligned} \tag{8}$$

$$\begin{aligned}
q_0 \rho_{m1\text{OPE}}^{\text{pert}}(q_0) &= \frac{q_0^5}{128\pi^4} \left\{ \rho_{m1\text{OPE}}^0\left(\frac{m_c^2}{q_0^2}\right) \left(1 + \frac{\alpha_s}{\pi} \ln \frac{\mu^2}{m_c^2}\right) \right. \\
&\quad \left. + \frac{\alpha_s}{\pi} \rho_{m1\text{OPE}}^1\left(\frac{m_c^2}{q_0^2}\right) \right\} \theta(q_0 - m_c),
\end{aligned} \tag{9}$$

$$\rho_{m1\text{OPE}}^0(z) = \frac{1}{4} - 2z + 2z^3 - \frac{1}{4}z^4 - 3z^2 \ln z, \tag{10}$$

$$\begin{aligned}
\rho_{m1\text{OPE}}^1(z) &= \frac{71}{48} - \frac{565}{36}z - \frac{7}{8}z^2 + \frac{625}{36}z^3 - \frac{109}{48}z^4 \\
&\quad - \left( \frac{49}{36} - \frac{116}{9}z + \frac{116}{9}z^3 - \frac{49}{36}z^4 \right) \ln(1-z) \\
&\quad + \left( \frac{1}{4} - \frac{17}{3}z - 11z^2 + \frac{113}{9}z^3 - \frac{49}{36}z^4 \right) \ln z \\
&\quad + \frac{2}{3} (1 - 8z + 8z^3 - z^4) \left( \text{Li}_2(z) + \frac{1}{2} \ln(1-z) \ln z \right) \\
&\quad - \frac{1}{3} z^2 (54 + 8z - z^2) \left( \text{Li}_2(z) - \zeta(2) + \frac{1}{2} \ln^2 z \right) \\
&\quad - 12z^2 \left( \text{Li}_3(z) - \zeta(3) - \frac{1}{3} \text{Li}_2(z) \ln(z) \right),
\end{aligned} \tag{11}$$

$$\begin{aligned}
q_0 \rho_{m1\text{OPE}}^{\text{cond}}(q_0) &= -\frac{m_c^2}{768\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle_m \\
&\quad \times \int_{\frac{m_c^2}{q_0^2}}^1 d\alpha \frac{(1-\alpha)^2}{\alpha^2} \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) \\
&\quad + \frac{q_0}{128\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle_m \int_{\frac{m_c^2}{q_0^2}}^1 d\alpha \alpha \theta(q_0 - m_c) \\
&\quad - q_0 \frac{\langle q^\dagger i D_0 q \rangle_m}{6\pi^2} \int_{\frac{m_c^2}{q_0^2}}^1 \alpha (1-\alpha) d\alpha \theta(q_0 - m_c) \\
&\quad + \frac{\langle q^\dagger i D_0 q \rangle_m}{6\pi^2} \int_0^1 d\alpha \alpha (1-\alpha) \frac{3}{2} q_0^2 \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) \\
&\quad + \frac{\langle \bar{q} q \rangle_m^2 + \langle q^\dagger q \rangle_m^2}{12} \delta(q_0 - m_c) \\
&\quad + \frac{1}{2^3 \cdot 3} \langle \bar{q} g \sigma G q \rangle_m \langle \bar{q} q \rangle_m \\
&\quad \times \left( \frac{1}{8} \left( \delta''(q_0 - m_c) - \frac{7}{m_c} \delta'(q_0 - m_c) \right) \right. \\
&\quad \left. + \frac{6}{m_c^2} \delta(q_0 - m_c) \right) \\
&\quad - q_0 \left[ \frac{q_0 \langle q^\dagger q \rangle_m}{4\pi^2} \int_{\frac{m_c^2}{q_0^2}}^1 d\alpha \alpha (1-\alpha) \theta(q_0 - m_c) \right. \\
&\quad + \frac{1}{8\pi^2} \left( \langle q^\dagger i D_0 i D_0 q \rangle_m + \frac{1}{12} \langle q^\dagger g_s \sigma G q \rangle_m \right) \\
&\quad \times \int_0^1 \alpha (1-\alpha) \left( \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) - \frac{\alpha q_0^3}{m_c^2} \delta'(q_0 - \frac{m_c}{\sqrt{\alpha}}) \right) d\alpha \\
&\quad - \frac{1}{96\pi^2} \langle q^\dagger g_s \sigma G q \rangle_m \int_0^1 \alpha (1-\alpha) \\
&\quad \times \left( 9\delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) + \frac{\alpha q_0^3}{m_c^2} \delta'(q_0 - \frac{m_c}{\sqrt{\alpha}}) \right) d\alpha \\
&\quad \left. + \frac{\langle q^\dagger g_s \sigma G q \rangle_m}{32\pi^2} \int_0^1 \alpha \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) d\alpha \right]
\end{aligned} \tag{12}$$

and

$$\rho_{m2\text{OPE}}(q_0, |\vec{q}|)|_{\vec{q}=0} = \rho_{m2\text{OPE}}^{\text{pert}}(q_0) + \rho_{m2\text{OPE}}^{\text{cond}}(q_0), \tag{13}$$

$$\begin{aligned}
\rho_{m2\text{OPE}}^{\text{pert}}(q_0) &= \frac{m_c q_0^4}{128\pi^4} \left\{ \rho_{m2\text{OPE}}^0\left(\frac{m_c^2}{q_0^2}\right) \left(1 + \frac{\alpha_s}{\pi} \ln \frac{\mu^2}{m_c^2}\right) \right. \\
&\quad \left. + \frac{\alpha_s}{\pi} \rho_{m2\text{OPE}}^1\left(\frac{m_c^2}{q_0^2}\right) \right\} \theta(q_0 - m_c),
\end{aligned} \tag{14}$$

$$\rho_{m2\text{OPE}}^0(z) = 1 + 9z - 9z^2 - z^3 + 6z(1+z) \ln z, \tag{15}$$

$$\begin{aligned}
\rho_{m2\text{OPE}}^1(z) = & 9 + \frac{665}{9}z - \frac{665}{9}z^2 - 9z^3 \\
& - \left( \frac{58}{9} + 42z - 42z^2 - \frac{58}{9}z^3 \right) \ln(1-z) \\
& + \left( 2 + \frac{154}{3}z - \frac{22}{3}z^2 - \frac{58}{9}z^3 \right) \ln z \\
& + \frac{8}{3}(1+9z-9z^2-z^3) \left( \text{Li}_2(z) + \frac{1}{2}\ln(1-z)\ln z \right) \\
& + z \left( 24 + 36z + \frac{4}{3}z^2 \right) \left( \text{Li}_2(z) - \zeta(2) + \frac{1}{2}\ln^2 z \right) \\
& + 24z(1+z) \left( \text{Li}_3(z) - \zeta(3) - \frac{1}{3}\text{Li}_2(z)\ln z \right), \tag{16}
\end{aligned}$$

$$\begin{aligned}
\rho_{m2\text{OPE}}^{\text{cond}}(q_0) = & -\frac{m_c}{768\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle_m \\
& \times \int_{\frac{m_c^2}{q_0^2}}^1 d\alpha \frac{(1-\alpha)^2}{\alpha} \frac{m_c}{\sqrt{\alpha}} \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) \\
& + \frac{m_c}{192\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle_m \int_{\frac{m_c^2}{q_0^2}}^1 d\alpha \frac{(1-\alpha)^3}{\alpha^2} \theta(q_0 - m_c) \\
& + \frac{m_c}{128\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle_m \int_{\frac{m_c^2}{q_0^2}}^1 d\alpha \theta(q_0 - m_c) \\
& + \frac{m_c \langle q^\dagger i D_0 q \rangle_m}{4\pi^2} \int_0^1 (1-\alpha) q_0 \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) d\alpha \\
& + \frac{\langle \bar{q} q \rangle_m^2 + \langle q^\dagger q \rangle_m^2}{12} \delta(q_0 - m_c) \\
& + \frac{1}{2^3 \cdot 3} \langle \bar{q} g \sigma G q \rangle_m \langle \bar{q} q \rangle_m \\
& \times \left( \frac{1}{8} \left( \delta''(q_0 - m_c) - \frac{3}{m_c} \delta'(q_0 - m_c) \right) \right. \\
& \quad \left. + \frac{3}{m_c^2} \delta(q_0 - m_c) \right) \\
& - q_0 \left[ m_c \frac{\langle q^\dagger q \rangle_m}{4\pi^2} \int_{\frac{m_c^2}{q_0^2}}^1 (1-\alpha) d\alpha \theta(q_0 - m_c) \right. \\
& + \frac{1}{4\pi^2} \left( \langle q^\dagger i D_0 i D_0 q \rangle_m + \frac{1}{12} \langle q^\dagger g_s \sigma G q \rangle_m \right) \\
& \times \int_0^1 2(1-\alpha) \left( \frac{\alpha q_0^2}{4m_c} \left( \frac{\sqrt{\alpha}}{m_c} \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) \right. \right. \\
& \quad \left. \left. + \delta'(q_0 - \frac{m_c}{\sqrt{\alpha}}) \right) \right) d\alpha \\
& - \frac{\langle q^\dagger g_s \sigma G q \rangle_m}{96\pi^2} \int_0^1 \left( 7\sqrt{\alpha} \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) \right. \\
& \quad \left. + \frac{\alpha q_0^2}{m_c} \delta'(q_0 - \frac{m_c}{\sqrt{\alpha}}) \right) d\alpha \\
& \left. + \frac{\langle q^\dagger g_s \sigma G q \rangle_m}{32\pi^2} \int_0^1 \sqrt{\alpha} \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) d\alpha \right] \tag{17}
\end{aligned}$$

and

$$\begin{aligned}
\rho_{m3\text{OPE}}(q_0, |\vec{q}|) |_{\vec{q}=0} = & -\frac{\langle q^\dagger q \rangle_m}{8\pi^2} \\
& \times \int_{\frac{m_c^2}{q_0^2}}^1 \alpha(1-\alpha) \left( q_0^2 - \frac{m_c^2}{\alpha} \right) d\alpha \theta(q_0 - m_c) \\
& - \frac{5}{8\pi^2} \left( \langle q^\dagger i D_0 i D_0 q \rangle_m + \frac{1}{12} \langle q^\dagger g_s \sigma G q \rangle_m \right) \\
& \times \int_0^1 \alpha(1-\alpha) q_0 \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) d\alpha \\
& + \frac{\langle q^\dagger g_s \sigma G q \rangle_m}{96\pi^2} \int_0^1 \alpha(1-\alpha) q_0 \delta(q_0 - \frac{m_c}{\sqrt{\alpha}}) d\alpha \\
& - \frac{\langle q^\dagger g_s \sigma G q \rangle_m}{32\pi^2} \int_{\frac{m_c^2}{q_0^2}}^1 \alpha d\alpha \theta(q_0 - m_c) \\
& + q_0 \frac{2\langle q^\dagger i D_0 q \rangle_m}{3\pi^2} \int_{\frac{m_c^2}{q_0^2}}^1 \alpha(1-\alpha) d\alpha \theta(q_0 - m_c), \tag{18}
\end{aligned}$$

where  $\text{Li}_s(z)$  is Polylogarithm and  $\zeta(s)$  is Riemann's zeta function. The matrix elements  $\langle \mathcal{O} \rangle_m$  stand for the expectation values of the operators  $\mathcal{O}$  in nuclear matter. We have used the factorization hypothesis for the four-quark and quark-gluon mixed operators in these equations and will discuss its validity in section IV. The dimension 7 condensate  $\langle \frac{\alpha_s GG}{\pi} \rangle_m \langle \bar{q} q \rangle_m$  term does not appear because its Wilson coefficient is equal to zero. The Wilson coefficients of  $\langle q^\dagger q \rangle_m$ ,  $\langle q^\dagger i D_0 q \rangle_m$ ,  $\langle q^\dagger i D_0 i D_0 q \rangle_m$ ,  $\langle q^\dagger g_s \sigma G q \rangle_m$  terms are different from those in the literature [28]. However, our results are consistent with the OPE of  $\Lambda$  [44] in the limit  $m_c^2 \rightarrow 0$ . Note that, in the case of  $\Lambda$ , extra contributions of  $\langle \bar{s}s \rangle_m$  condensate appear and the Wilson coefficients of the gluon condensate are modified.

The values of the parameters in OPE are summarized below. In the linear density approximation, which is valid at sufficiently low density [17, 45], the expectation values of the condensates in nuclear matter  $\langle \mathcal{O} \rangle_m$  are given as

$$\begin{aligned}
\langle \bar{q} q \rangle_m &= \langle \bar{q} q \rangle_0 + \rho \langle \bar{q} q \rangle_N = \langle \bar{q} q \rangle_0 + \rho \frac{\sigma_N}{2m_q} \\
\langle \bar{s}s \rangle_m &= \langle \bar{s}s \rangle_0 + \rho \langle \bar{s}s \rangle_N \\
\langle q^\dagger q \rangle_m &= \rho \frac{3}{2} \\
\langle \frac{\alpha_s}{\pi} G^2 \rangle_m &= \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 + \rho \langle \frac{\alpha_s}{\pi} G^2 \rangle_N \\
\langle q^\dagger i D_0 q \rangle_m &= \rho \langle q^\dagger i D_0 q \rangle_N = \rho \frac{3}{8} M_N A_2^q \\
\langle \bar{q} i D_0 q \rangle_m &= m_q \langle q^\dagger q \rangle_m \simeq 0 \\
\langle q^\dagger g \sigma \cdot G q \rangle_m &= \rho \langle q^\dagger g \sigma \cdot G q \rangle_N \\
\langle q^\dagger i D_0 i D_0 q \rangle_m + \frac{1}{12} \langle q^\dagger g \sigma \cdot G q \rangle_m \\
&= (\langle q^\dagger i D_0 i D_0 q \rangle_N + \frac{1}{12} \langle q^\dagger g \sigma \cdot G q \rangle_N) \rho \\
&= \rho \frac{1}{4} M_N^2 A_3^q \tag{19}
\end{aligned}$$

parameters	values
$\langle \bar{q}q \rangle_0$	$-(0.246 \pm 0.002 \text{GeV})^3$ [48]
$m_q$	4.725 MeV [49]
$\sigma_N$	45 MeV
$\langle q^\dagger q \rangle_m$	$\rho_2^{\frac{3}{2}}$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle_0$	$0.012 \pm 0.0036 \text{GeV}^4$ [50]
$\langle \frac{\alpha_s}{\pi} G^2 \rangle_N$	$-0.65 \pm 0.15 \text{GeV}^4$ [51]
$A_2^q$	$0.62 \pm 0.06$ [52]
$A_3^q$	$0.15 \pm 0.02$ [52]
$\langle q^\dagger g \sigma \cdot G q \rangle_N$	$-0.33 \text{GeV}^2$ [51]
$m_c$	$1.67 \pm 0.07 \text{GeV}$ [49]
$\alpha_s$	0.5

TABLE I: Values of parameters appearing in Eq. (19).

where  $\langle \mathcal{O} \rangle_0$  and  $\langle \mathcal{O} \rangle_N$  are, respectively, the vacuum and nucleon expectation values of the operator  $\mathcal{O}$ , and  $\langle \bar{q} \cdots q \rangle$  and  $\langle q^\dagger \cdots q \rangle$  denotes the average over the up and down quarks,  $\frac{1}{2} (\langle \bar{u} \cdots u \rangle + \langle \bar{d} \cdots d \rangle)$  and  $\frac{1}{2} (\langle u^\dagger \cdots u \rangle + \langle d^\dagger \cdots d \rangle)$ , respectively. The quantities  $A_2^q$  and  $A_3^q$  can be expressed as moments of the parton distribution functions [38]. For the quark condensate  $\langle \bar{q}q \rangle$ , higher order density terms have been calculated by chiral perturbation theory [46, 47]. However, their contributions are small up to the normal nuclear matter density and thus we do not take them into account in this study. The numerical values of the parameters appearing in Eq. (19) are given in Table I.

### B. Phenomenological side of the $\Lambda_c$ correlation function

The correlation function at the physical energy region ( $q^2 > 0$ ) can be described by the hadronic degrees of freedom. In this study, we use the so called “pole + continuum” ansatz for the correlator in such energy region. The pole stands for the contributions of the ground state and is assumed to be proportional to the single  $\Lambda_c$  baryon propagator  $G(q)$  in vacuum,

$$G(q) = \frac{\not{q} + M_{\Lambda_c}}{(q^2 - M_{\Lambda_c}^2 + i\epsilon)}. \quad (20)$$

Then the pole contribution in the spectral function  $\rho^T(q) \equiv \frac{1}{\pi} \text{Im}[\Pi^T(q)]$  is

$$\begin{aligned} & -\frac{1}{\pi} \text{Im} \left[ |\lambda^2| \frac{\not{q} + M_{\Lambda_c}}{(q^2 - M_{\Lambda_c}^2 + i\epsilon)} \right] \\ & = |\lambda^2| (\not{q} + M_{\Lambda_c}) \delta(q^2 - M_{\Lambda_c}^2). \end{aligned} \quad (21)$$

Here,  $|\lambda^2|$  is the residue and gives the coupling strength between the ground state and the employed interpolating operator. Taking into account the contributions of the

lowest lying negative parity state  $\Lambda_c^-$ , the phenomenological side of the spectral function in the “pole + continuum” ansatz can be expressed as

$$\begin{aligned} \rho^T(q) &= |\lambda_+^2| (\not{q} + M_{\Lambda_c}) \delta(q^2 - M_{\Lambda_c}^2) \\ &+ |\lambda_-^2| (\not{q} - M_{\Lambda_c^-}) \delta(q^2 - M_{\Lambda_c^-}^2) \\ &+ \frac{1}{\pi} \text{Im} [\Pi_{\text{OPE}}(q)] \theta(q^2 - q_{th}^2). \end{aligned} \quad (22)$$

For the continuum states, the quark hadron duality is assumed.

In nuclear matter, the  $\Lambda_c$  propagator can be described as

$$G(q_0, |\vec{q}|) = \frac{Z'(q_0, |\vec{q}|)}{\not{q} - M - \Sigma(q_0, |\vec{q}|) + i\epsilon} \quad (23)$$

where  $\Sigma(q_0, |\vec{q}|)$  is the self-energy and  $Z'(q_0, |\vec{q}|)$  denotes the renormalization factor of the wave function. As in Eq. (4), the self-energy can be decomposed as

$$\begin{aligned} \Sigma(q_0, |\vec{q}|) &= \Sigma^{s'}(q_0, |\vec{q}|) + \Sigma^{v'}(q_0, |\vec{q}|) \not{q} \\ &+ \Sigma^{q'}(q_0, |\vec{q}|) \not{q}. \end{aligned} \quad (24)$$

Renormalizing the  $\Sigma^{q'}(q_0, |\vec{q}|)$  contributions to the  $Z'(q_0, |\vec{q}|)$ ,  $\Sigma^{v'}(q_0, |\vec{q}|)$  and effective mass  $M_{\Lambda_c}^*$ , the in-medium  $\Lambda_c$  baryon propagator  $G(q_0, |\vec{q}|)$  is expressed as

$$\begin{aligned} G(q_0, |\vec{q}|) &= Z(q_0, |\vec{q}|) \frac{\not{q} - \not{q} \Sigma_v + M_{\Lambda_c}^*}{(q_0 - E_{\Lambda_c} + i\epsilon)(q_0 + \bar{E}_{\Lambda_c} - i\epsilon)}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} E_{\Lambda_c} &= \Sigma_v + \sqrt{M_{\Lambda_c}^{*2} + \vec{q}^2}, \\ \bar{E}_{\Lambda_c} &= -\Sigma_v + \sqrt{M_{\Lambda_c}^{*2} + \vec{q}^2}. \end{aligned} \quad (26)$$

The contribution of the positive energy state in  $G(q_0, |\vec{q}|)$  is

$$Z(q_0, |\vec{q}|) \frac{1}{2\sqrt{M_{\Lambda_c}^{*2} + \vec{q}^2}} \frac{\gamma_0 E_{\Lambda_c} - \not{q} \Sigma_v + M_{\Lambda_c}^*}{(q_0 - E_{\Lambda_c} + i\epsilon)}. \quad (27)$$

After taking the rest frame of nuclear matter, the forward-time spectral function in nuclear matter can be described as

$$\begin{aligned} \rho(q_0, |\vec{q}| = 0) &= \frac{|\lambda_+^2|}{2M_{\Lambda_c}^*} (\gamma_0 E_{\Lambda_c} - \not{q} \Sigma_v + M_{\Lambda_c}^*) \delta(q_0 - E_{\Lambda_c}) \\ &+ \frac{|\lambda_-^2|}{2M_{\Lambda_c^-}^*} (\gamma_0 E_{\Lambda_c^-} - \not{q} \Sigma_v - M_{\Lambda_c^-}^*) \delta(q_0 - E_{\Lambda_c^-}) \\ &+ \frac{1}{\pi} \text{Im} [\Pi_{\text{OPE}}(q_0, |\vec{q}| = 0)] \theta(q_0 - q_{th}). \end{aligned} \quad (28)$$

### C. Equation of $\Lambda_c$ QCD sum rule with parity projection

As we have introduced in Eq.(6), the OPE description of Eq.(7) and the phenomenological description of Eq.(28) can be connected with the help of the analyticity of the correlation function in  $q_0$  plane. For specifying the kernel  $W(q_0)$ , we use the Gaussian sum rule and its equation is given as

$$G_{mOPE}^\pm(\tau) = G_{SPF}^\pm(\tau), \quad (29)$$

where

$$G_{SPF}^\pm(\tau) = \int_0^\infty \rho^\pm(q_0) \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) dq_0 \quad (30)$$

and

$$G_{mOPE}^\pm(\tau) \equiv G_{m1OPE}(\tau) \pm G_{m2OPE}(\tau) + G_{m3OPE}(\tau) \quad (31)$$

with

$$\begin{aligned} G_{m1OPE}(\tau) &= \int_0^\infty q_0 \rho_{m1OPE}(q_0, |\vec{q}|) \\ &\quad \times \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) dq_0 \\ G_{m2OPE}(\tau) &= \int_0^\infty \rho_{m2OPE}(q_0, |\vec{q}|) \\ &\quad \times \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) dq_0 \\ G_{m3OPE}(\tau) &= \int_0^\infty \rho_{m3OPE}(q_0, |\vec{q}|) \\ &\quad \times \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) dq_0. \end{aligned} \quad (32)$$

Here  $\rho^\pm(q_0)$  is the hadronic spectral function and  $\pm$  stands for the parity of the hadronic states,

$$\begin{aligned} \rho^\pm(q_0) &= |\lambda^\pm|^2 \delta(q_0 - E_{\Lambda_c^\pm}) \\ &\quad + \frac{1}{\pi} \text{Im} [\Pi_{mOPE}^\pm(q_0)] \theta(q_0 - q_{th}^\pm). \end{aligned} \quad (33)$$

Generally, the kernel of the Gaussian sum rule is  $\frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - s)^2}{4\tau}\right)$  and contains two parameters,  $\tau$  and  $s$ . We set the value of  $s$  to  $m_c^2$ , and as a result, the exponential suppression starts from  $m_c$ . This is the different point from the case of the Borel sum rule where the kernel,  $\exp\left(-\frac{q_0^2}{M^2}\right)$ , is used. One naively expects that the Gaussian sum rule strongly reflects on the behavior of the spectral function above  $m_c$  more than the Borel sum rule. It is one advantage of using the Gaussian sum rule.

We extract the values of parameters in  $\rho^\pm(q_0)$  from Eq.(29) by minimizing  $\chi^2$ , defined as

$$\chi^2 = \frac{1}{n_{set} \times n_\tau} \sum_{j=1}^{n_{set}} \sum_{i=1}^{n_\tau} \frac{(G_{mOPE}^j(\tau_i) - G_{SPF}^j(\tau_i))^2}{\sigma^j(\tau_i)^2}, \quad (34)$$

where the errors of  $G_{mOPE}$ ,  $\sigma(\tau_i)$ , are evaluated based on the method proposed in Ref. [53]:

$$\sigma^j(\tau_i)^2 = \frac{1}{n_{set} - 1} \sum_{j=1}^{n_{set}} (G_{mOPE}^j(\tau_i) - \bar{G}_{mOPE}(\tau_i))^2, \quad (35)$$

with

$$\bar{G}_{mOPE}(\tau_i) = \frac{1}{n_{set}} \sum_{j=1}^{n_{set}} G_{mOPE}^j(\tau_i). \quad (36)$$

Here  $n_\tau$  and  $n_{set}$  are the numbers of the point  $\tau$  in the analyzed  $\tau$  region and of the condensate sets which are randomly generated with errors, respectively.

The parameter region of  $\tau$  is chosen as follows. The lowest value of  $\tau$  is determined by the convergence of the OPE while the highest value of  $\tau$  is constrained to satisfy the pole dominance condition. These conditions will justify the truncation of the OPE. Specifically, we use the well established criterion that the ratio of the highest dimensional term to the whole  $G_{mOPE}$  is less than 0.1 and the ratio of the pole contribution to the whole  $G_{mSPF}$  is more than 0.5. The region  $1.25 < \tau[\text{GeV}^4] < 3.5$  satisfies the above conditions in vacuum. We use the same parameter region for the analyses in nuclear matter.

## III. RESULTS OF $\Lambda_c$

### A. Behavior of OPE

We first discuss the behaviors of  $G_{mOPE}^\pm(\tau)$  in vacuum, shown in Fig. 1. We find that the OPE of the positive parity correlation function is much larger than that of the negative parity state. In the hadronic degrees of freedom, this means that the interpolating operator  $J_{\Lambda_c}$  couples mainly to the positive parity ground states, and that the structure of  $\Lambda_c$  is similar to the structure of the interpolating operator. This result is consistent with the quark model picture where the angular momentum between the quarks are all S-wave. On the other hand, the present interpolating operator hardly couples to the negative parity states.

$G_{mOPE}^+(\tau)$  in vacuum and at normal nuclear matter density are shown in Fig. 2. The LO-perturbative, the NLO-perturbative, the four-quark and the vector quark condensate  $\langle q^\dagger q \rangle$  terms whose contributions are large, are also shown in the same figure. We find that the  $\alpha_s$  correction on the perturbative terms are as large as twice of the leading order term. Therefore, it is important to include the  $\alpha_s$  corrections. Among the non-perturbative

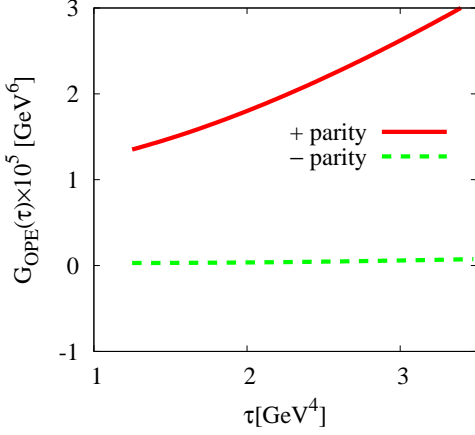


FIG. 1: The positive and negative parity OPE,  $G_{mOPE}^{\pm}(\tau)$ , in vacuum.

contributions, the four-quark condensate is dominant in vacuum. Fig. 2 shows that the in-medium modification of the four-quark condensate and  $\langle q^{\dagger}q \rangle_m$  is the main origin of the density dependence of  $G_{mOPE}^{\pm}(\tau)$ . Therefore, we find that the charm quark plays less role than the light quarks as far as the density dependence is concerned. We use the factorization hypothesis for the four-quark condensate in this section. Another possible density dependence of the four-quark condensate and its effect to the result will be discussed in section IV. It is important to notice that the  $\langle \bar{q}q \rangle$  term does not appear in the correlation function due to the structure of the interpolating field  $J_{\Lambda_c}$ . The di-quark structure in  $J_{\Lambda_c}$  can be described as  $(q^{Ta}C\gamma_5q^b) = (-q_L^{Ta}C\gamma_5q_L^b + q_R^{Ta}C\gamma_5q_R^b)$  by using the left and the right handed spinors, and thus the quark condensate  $\langle \bar{q}q \rangle$  where a left-handed spinor is paired to a right-handed one does not have contributions in the chiral limit. Thus  $\Lambda_c$  feels the in-medium effects mainly through the four-quark condensate and  $\langle q^{\dagger}q \rangle_m$ .

### B. Total energy

We investigate the hadron properties by minimizing  $\chi^2$ . In the case of the positive parity state, it is found that various combinations of the parameters lead the value of  $\chi^2$  to vicinity of 1. The trivial solutions are contained in these combinations. The values that are close to  $E_{\Lambda_c} = m_c$  and  $q_{th} = m_c$  give small values of  $\chi^2 - 1$  because, except for the terms of the dimension 8 condensate and the part of the gluon condensate,  $G_{mOPE}(\tau)$  can be expressed by  $\delta(q_0 - m_c)$  and  $\theta(q_0 - q_{th})$ . These values do not correspond to physical solutions and must be discarded. In order to avoid such solutions, we impose further conditions. The interpolating operator  $J_{\Lambda_c}$  for  $\Lambda_c$  couples to the continuum states starting from around the value of the lowest threshold,  $\Sigma_c\pi$ , i.e., 2600

$q_{th}$ [GeV]	2.6	2.7	2.715	2.8	2.9	3.0	3.1
$E_{\Lambda_c}$ [GeV]	2.218	2.275	2.285	2.320	2.370	2.397	2.419
$ \lambda ^2$	1.20	1.55	1.62	1.97	2.40	2.77	3.08
$\chi^2$	1.002	1.007	1.009	1.020	1.046	1.078	1.116

TABLE II: The  $q_{th}$  dependence of the result.

MeV. Therefore, we exclude the  $q_{th}$  parameter region,  $q_{th} < 2600$  MeV. The  $q_{th}$  dependence of the result in vacuum are summarized in Table II. We find that the uncertainty of the calculated ground state mass is about 100 MeV and the choice of  $q_{th} = 2.715$  GeV can reproduce the experimental value. Therefore, we will fix the value of  $q_{th}$  as 2.715 GeV and apply the analyses to the  $\Lambda_c$  baryon in nuclear matter. The density dependence of the energy  $E_{\Lambda_c}$  is shown in Fig. 3. We find that the energy of  $\Lambda_c$  in nuclear matter, namely the peak position in the spectral function, increases as the density increases. At the normal nuclear matter density, its value is 2.385 GeV. This result indicates that  $\Lambda_c$  feels a net repulsive potential in nuclear matter. The behavior is different from the nucleon and  $\Lambda$ , whose total energy gradually decreases and causes the formation of bound states in nuclei. As for the negative parity state, the analysis does not work well due to the small coupling between the present interpolating field and  $\Lambda_c^-$  state.

### C. Effective mass and vector self-energy

The analyses in the previous subsection show that the energy of  $\Lambda_c$  in nuclear matter increases. In relativistic phenomenological models, the total energy can be decomposed into the effective mass  $m_B^*$  and the vector self-energy  $\Sigma_B^v$  as we have introduced in Eq. (26). These quantities have been investigated from the previous QCD sum rule analyses for the nucleon [17, 51, 54–62] and  $\Lambda$  [38, 44, 62–64]. The studies show that significant cancellations of the modifications of  $m_B^*$  and  $\Sigma_B^v$  for both the nucleon and  $\Lambda$  cases occurs. The modifications of  $m_B^*$  and  $\Sigma_B^v$  can be connected with the coupling strength of the baryon  $B$  to the scalar and the vector mean fields, respectively. In a naive estimation where exchanged light mesons only couples to the light quarks in the baryon, one can derive a simple relation between the coupling strengths of the nucleon and that of  $\Lambda$ . If this approximation is valid, the effective masses and the vector self-energies satisfy the following relation:  $M_{\Lambda}^* - M_{\Lambda} \approx \frac{2}{3}(M_N^* - M_N)$  and  $\Sigma_{\Lambda}^v \approx \frac{2}{3}\Sigma_N^v$ . However, the previous QCD sum rule analyses show a large violation of the above relation [38, 44, 62–64]. We will investigate the effective mass and the vector self-energy of  $\Lambda_c$  and compare the results with those of the nucleon and  $\Lambda$ .

As we see from Eq. (33), we can study the energy  $E_{\Lambda_c^{\pm}}$  in the parity projected QCD sum rule. Here we parametrize the phenomenological side of each

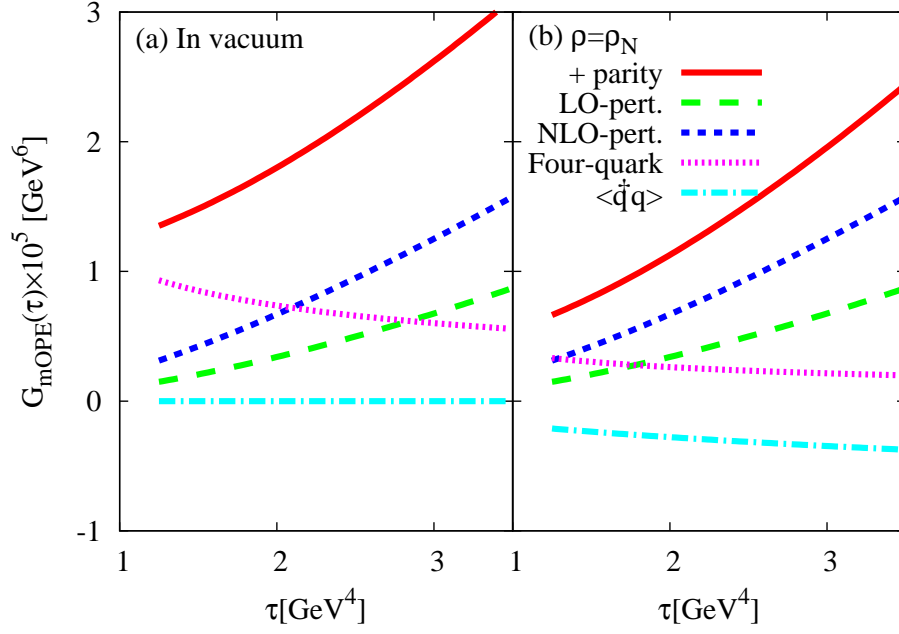


FIG. 2: The density dependences of the positive parity OPE  $G_{mOPE}^+(\tau)$ . The LO-perturbative, the NLO-perturbative, the four-quark and the vector quark condensate  $\langle \bar{q}q \rangle$  terms are shown. Here,  $\rho_N$  means normal nuclear matter density.

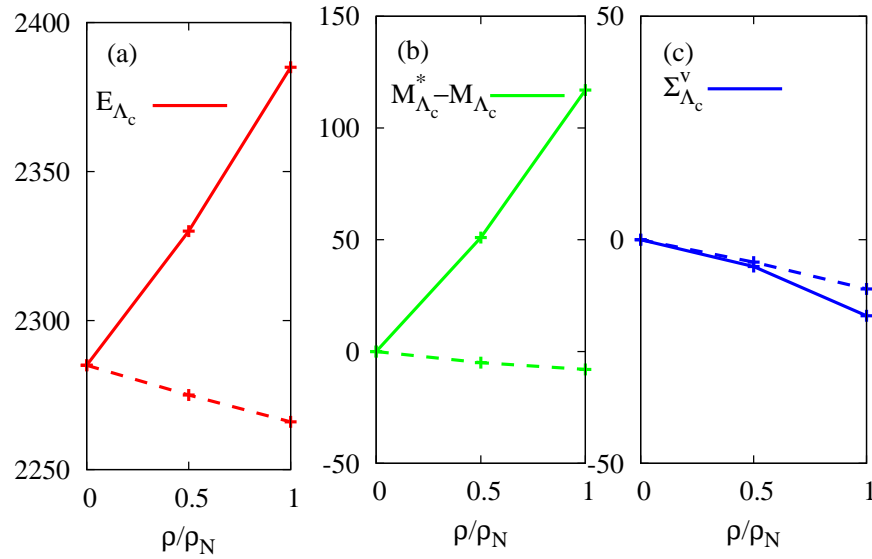


FIG. 3: The density dependences of (a) the energy  $E_{\Lambda_c}$  [MeV], (b) the effective mass  $M_{\Lambda_c}^*$  [MeV] and (c) the vector self-energy  $\Sigma_{\Lambda_c}^v$  [MeV]. The solid lines show the results with the density dependence of the four-quark condensate according to the factorization hypothesis while dashed lines is the case where its density dependence are estimated from the perturbative chiral quark model.



$G_{miOPE}(\tau)$  ( $i = 1, 2, 3$ ) and investigate the effective mass and the vector self-energy. The specific forms of phenomenological sides are as follows:

$$\begin{aligned}
G_{m1SPF}(\tau) &= \int_0^\infty dq_0 \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \\
&\quad \times \left( |\lambda^+|^2 \frac{M_{\Lambda_c}}{2\sqrt{M_{\Lambda_c}^{*2}}} \delta(q_0 - E_{\Lambda_c}) \right. \\
&\quad \left. + \frac{1}{\pi} \text{Im} [\Pi_{m1OPE}(q_0)] \theta(q_0 - q_{th}) \right) \\
G_{m2SPF}(\tau) &= \int_0^\infty dq_0 \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \\
&\quad \times \left( |\lambda^+|^2 \frac{M_{\Lambda_c}^*}{2\sqrt{M_{\Lambda_c}^{*2}}} \delta(q_0 - E_{\Lambda_c}) \right. \\
&\quad \left. + \frac{1}{\pi} \text{Im} [\Pi_{m2OPE}(q_0)] \theta(q_0 - q_{th}) \right) \\
G_{m3SPF}(\tau) &= \int_0^\infty dq_0 \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \\
&\quad \times \left( |\lambda^+|^2 \frac{-\Sigma_{\Lambda_c}^v}{2\sqrt{M_{\Lambda_c}^{*2}}} \delta(q_0 - E_{\Lambda_c}) \right. \\
&\quad \left. + \frac{1}{\pi} \text{Im} [\Pi_{m3OPE}(q_0)] \theta(q_0 - q_{th}) \right), \tag{37}
\end{aligned}$$

where the structure of the lowest lying pole comes from the in-medium propagators of  $\Lambda_c^\pm$  of Eq. (25). We do not consider the contribution of negative parity state  $\Lambda_c^-$  in the above equation because its contribution are quite small. The validity of such treatment in vacuum are guaranteed by the fact that  $G_{mOPE}^-(\tau)$  is much smaller than the  $G_{mOPE}^+(\tau)$ . Its validity in nuclear matter can be also investigated by comparing  $G_{m2SPF}(\tau)$  with  $G_{m2OPE}(\tau)$ .  $G_{m2SPF}(\tau)$  is characterized by  $q_{th}$ ,  $|\lambda^+|^2$  and  $E_{\Lambda_c}$  whose values have been already obtained from the analyses of the parity projected QCD sum rules. We confirm that such treatment is valid up to the normal nuclear matter density. By fitting the  $G_{m1SPF}(\tau)$ ,  $G_{m3SPF}(\tau)$  and corresponding functions in the OPE side, we investigate the effective mass and the vector self-energy. These density dependences are shown in Fig. 3. From this figure, we find that the effective mass increases while the vector self-energy decreases as the density increases. At the normal nuclear matter density, the values of the effective mass and the vector self-energy is 2.402 GeV and -0.017 GeV, respectively. Comparing them with the previous  $\Lambda_c$  QCD sum rule analysis with the same interpolating operator [28], we agree in the increase of the effective mass but the sign of the vector self-energy is different. The discrepancy may come from the first order  $\alpha_s$  corrections and the dimension 8 condensate. The small contribution of the dimension 8 condensate implies the good

convergence of the OPE and thus we can use  $G_{mOPE}(\tau)$  in lower energy region which contains much information about the  $\Lambda_c$  ground state. In another preceding study where a different interpolating operator is used [29], both the signs of the modifications of the effective mass and the vector self-energies are opposite to our results and the magnitude of their values are much larger than ours.

Comparing our results with the previous QCD sum rule of the nucleon [62] and  $\Lambda$  [44], we find that the shift of the effective mass is about  $M_{\Lambda_c}^* - M_{\Lambda_c} \approx -(M_{\Lambda}^* - M_{\Lambda}) \approx -\frac{1}{3}(M_N^* - M_N)$  and the vector self-energy is about  $\Sigma_{\Lambda_c}^v \approx -\frac{1}{3}\Sigma_{\Lambda}^v \approx -\frac{1}{12}\Sigma_N^v$ . Their values of  $\Lambda_c$  are quite different. The smallness of the vector self-energy may be understood from the contribution of the  $\langle q^\dagger q \rangle$  term in  $G_{m3OPE}(\tau)$ . In the case of the nucleon and  $\Lambda$  analyses [51, 59–62], this contribution plays a dominant role to determine the vector self-energy. The  $\langle q^\dagger q \rangle$  contribution gives a repulsive vector self-energy of  $\Lambda_c$  as well as of the nucleon and  $\Lambda$ . However, the contribution in  $\Lambda_c$  is small because the Wilson coefficient of this term is proportional to  $\frac{(q_0^2 - m_c^2)^4}{(q_0^2)^3}$  and its value becomes small as  $q_0$  goes close to  $m_c$ . Therefore, when we investigate  $\Lambda_c$  whose energy is close to charm quark mass, the contribution of  $\langle q^\dagger q \rangle$  to the vector self-energy is suppressed. This suppression implies that the coupling strength of the vector field is affected by not only the light quarks but also the heavy quark because the Wilson coefficient contains both the light and the heavy quark propagators.

#### IV. DISCUSSION

We have investigated the density dependence of the energy, the effective mass and the vector self-energy of  $\Lambda_c$ . The results are different from our naive expectation and the qualitative behaviors of the nucleon and  $\Lambda$ . Here we discuss the density dependence of the four-quark condensate and its effects to the results of the analyses. As we have shown in Fig. 2, the four-quark condensate is dominant and thus will strongly affect the density dependence of  $G_{mOPE}(\tau)$ . We also investigate the in-medium modification of  $\Lambda$  and  $\Lambda_b$  in the same framework of our QCD sum rule analyses.

##### A. Density dependence of four-quark condensate

In this subsection, we discuss the density dependence of the four-quark condensate and its effect to the in-medium modifications of  $\Lambda_c$ . We specifically consider two types of density dependences. The first one is based on the factorization hypothesis and the second is estimated from the perturbative chiral quark model (PCQM) [75, 76].

The in-medium condensates are usually evaluated in the linear density approximation and can be described as  $\langle \mathcal{O} \rangle_m \cong \langle \mathcal{O} \rangle_0 + \rho \langle \mathcal{O} \rangle_N$ . It is important to obtain pre-

cise values of  $\langle \mathcal{O} \rangle_N$ , the values of the condensates in the nucleon. In some cases, they can be experimentally subtracted. For instance, expectation values of some twist-4 four-quark condensates in the nucleon can be estimated from deep inelastic scattering data [44, 65, 66]. However, general four-quark condensates can not be determined. A commonly used technique is the factorization hypothesis, where a four-quark condensate is given by the product of two quark condensates as

$$\langle u_\alpha^a \bar{u}_\beta^b d_\gamma^c \bar{d}_\delta^d \rangle \rightarrow \langle u_\alpha^a \bar{u}_\beta^b \rangle \langle d_\gamma^c \bar{d}_\delta^d \rangle, \quad (38)$$

where  $a, b, c$  and  $d$  are color indices and  $\alpha, \beta, \gamma$  and  $\delta$  are spinor indices. The factorization can only be justified in the large  $N_c$  limit [14, 15] and the validity in  $N_c = 3$  is not so clear. In fact, there are some studies which claim significant violation of the factorization in the QCD sum rule analyses of  $\rho$  meson [67–71],  $\phi$  meson [70, 72] and the nucleon [58, 62, 73–75].

We here consider a model dependent but more sophisticated approach based on PCQM. In general, the four-quark condensates which appear in the OPE calculations have different density dependences. The four-quark condensates can be expressed as linear combinations of the independent four-quark condensates and their density dependence are different from each other. In Refs. [75, 76], the nucleon expectation values of the various independent four-quark condensates are estimated by PCQM. We estimate the density dependence of the four-quark condensate from these results and reinvestigate the in-medium modification of  $\Lambda_c$ .

In the case of the interpolating operator  $J_{\Lambda_c}$ , the structure of the four-quark condensate can be described as

$$\begin{aligned} & \langle (\epsilon^{abc} u^a C \gamma_5 d^b) \cdot (\epsilon^{efc} \bar{d}^f \gamma_5 C \bar{u}^e) \rangle_m \\ &= -\frac{1}{4} \left[ \langle \bar{d}^f d^b \bar{u}^e u^a \rangle_m + \langle \bar{d}^f \gamma_5 d^b \bar{u}^e \gamma_5 u^a \rangle_m \right. \\ & \quad - \frac{1}{2} \langle \bar{d}^f \sigma_{\mu\nu} d^b \bar{u}^e \sigma^{\mu\nu} u^a \rangle_m + \langle \bar{d}^f \gamma_\mu d^b \bar{u}^e \gamma^\mu u^a \rangle_m \\ & \quad \left. + \langle \bar{d}^f \gamma_5 \gamma_\mu d^b \bar{u}^e \gamma_5 \gamma^\mu u^a \rangle_m \right] \epsilon^{abc} \epsilon^{efc}. \end{aligned} \quad (39)$$

Here, we have decomposed the four-quark condensate into independent four-quark condensates which have the different Lorentz structures. We find that the twist-0 four-quark condensates only appear in this sum rule. In the case of the factorization hypothesis, the density dependence of the four-quark condensate is expressed as

$$\begin{aligned} & \langle (\epsilon^{abc} u^a C \gamma_5 d^b) \cdot (\epsilon^{efc} \bar{d}^f \gamma_5 C \bar{u}^e) \rangle_m \\ &= -\frac{1}{6} (\langle \bar{d} d \rangle_m \langle \bar{u} u \rangle_m + \langle d^\dagger d \rangle_m \langle u^\dagger u \rangle_m) \\ &= -\frac{1}{6} (\langle \bar{q} q \rangle_m^2 + \langle q^\dagger q \rangle_m^2) \\ &= -\frac{1}{6} \left( \langle \bar{q} q \rangle_0^2 + \rho \frac{\sigma_N}{m_q} \langle \bar{q} q \rangle_0 + \left( \frac{\sigma_N^2}{4m_q^2} + \frac{9}{4} \right) \rho^2 \right), \end{aligned} \quad (40)$$

where the isospin symmetry is used. This density dependence is used in the previous sections. On the other hand, from the results of the PCQM calculations [75, 76], the density dependence of whole four-quark condensate can be written as

$$\begin{aligned} & \langle (\epsilon^{abc} u^a C \gamma_5 d^b) \cdot (\epsilon^{efg} \bar{d}^f \gamma_5 C \bar{u}^e) \rangle_m \\ &= -\frac{1}{6} \langle \bar{q} q \rangle_0^2 - \frac{1}{4} 0.935 \langle \bar{q} q \rangle_0 \rho + \mathcal{O}(\rho^2). \end{aligned} \quad (41)$$

The coefficient of  $\langle \bar{q} q \rangle_0^2$  is determined by the factorization hypothesis. For simplicity, we call the density dependences of Eq. (40) and Eq. (41) as F-type and QM-type, respectively. Comparing Eq. (40) with Eq. (41), we find that the QM-type in-medium modification is much weaker than that of the F-type. Using this density dependence, we recalculate the in-medium modifications of  $\Lambda_c$ . The results are shown as dashed lines in Fig. 3. At the normal nuclear matter density, the values of the energy, the effective mass and the vector self-energy are 2.266 GeV, 2.277 GeV and -0.011 GeV, respectively. The total energy decreases about 20 MeV, which implies that  $\Lambda_c$  feels a net attractive potential and forms bound states in nuclear matter. The results indicate that the density dependence of the four-quark condensate strongly affects the in-medium modifications of  $\Lambda_c$ . Therefore, in turn,  $\Lambda_c$  is useful as a probe of the density dependence of the four-quark condensate.

Finally, we comment on the relation between the partial restoration of chiral symmetry and the four-quark condensate of Eq. (39). The chiral condensate is usually considered as an order parameter of the chiral transition, but the role of four-quark condensates in the spontaneous breaking of the chiral symmetry is still an open issue. The effects from four-quark condensates to the phase transition are discussed in Refs. [77–80]. However, the four-quark condensate of Eq. (39) is singlet under the chiral  $SU(2) \times SU(2)$  transformation and thus its in-medium modification is not directly related to the partial restoration of chiral symmetry. The knowledge of this density dependence may be useful when we investigate other hadrons in nuclear matter. Some kinds of four-quark condensates which appear in OPE may contain the four quark condensate of Eq. (39).

## B. Analyses of $\Lambda$ and $\Lambda_b$

We carry out the analyses of  $\Lambda$  and  $\Lambda_b$  in this subsection. The OPE representation of the  $\Lambda_b$  correlation function is same as that of  $\Lambda_c$  except for the value of the quark mass  $m_Q$ . On the other hand, the OPE of  $\Lambda$  contains some extra terms in which the strange quark condensate is treated differently from the charm quark. The Wilson coefficients of the gluon condensate and strange quark condensate terms are different from the case of  $\Lambda_c$ . Therefore, we refer Ref. [44] and construct the parity projected Gaussian sum rule of  $\Lambda$ . The values of

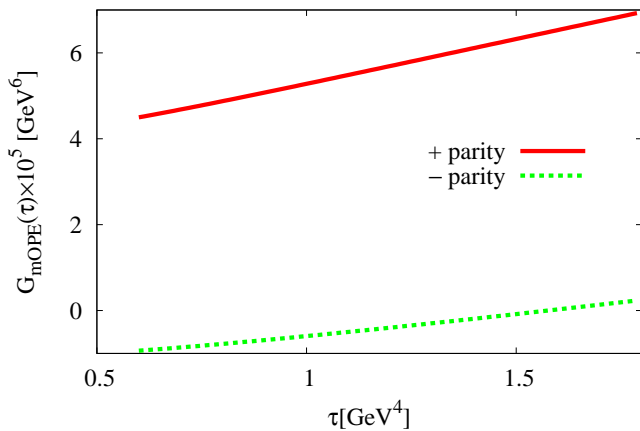


FIG. 4: The positive and the negative parity OPE  $G_{mOPE}^{\pm}(\tau)$  of  $\Lambda$  in vacuum.

the new parameters are as follows:  $m_b = 4.78 \pm 0.06$  GeV [49],  $m_s = 130 \pm 8$  MeV [49],  $\langle \bar{s}s \rangle_0 = 0.8 \langle \bar{q}q \rangle_0$  [44] and  $\langle \bar{s}s \rangle_N = 0.1 \langle \bar{q}q \rangle_N$  [44]. The analyzed parameter region for  $\Lambda$  and  $\Lambda_b$  are  $0.6 < \tau[\text{GeV}^4] < 1.8$  and  $9.5 < \tau[\text{GeV}^4] < 23$ , respectively, which are determined by the same criterion as the  $\Lambda_c$  analyses.

The behavior of the OPE for  $\Lambda$  is shown in Fig. 4. We find that the interpolating operator couples to both the positive and negative parity states, in contrast to the case of  $\Lambda_c$ . The result indicates that we should take into account effects of the negative parity state when investigating the effective mass and the vector self-energy of the positive parity state. Therefore, we leave the individual quantities for a future work and investigate the density dependence only of the energy of the positive parity state in this study. The qualitative behavior of  $G_{mOPE}(\tau)$  of  $\Lambda_b$  is same as that of  $\Lambda_c$ . The results of the analyses of  $\Lambda$  and  $\Lambda_b$  are shown in Figs. 5 and 6. The values of the threshold parameter is fixed to  $q_{th}^{\Lambda} = 1.52$  GeV and  $q_{th}^{\Lambda_b} = 6.03$  GeV, respectively. They are taken so as to reproduce the experimental mass in vacuum.

Fig. 5 shows that energy of  $\Lambda$  increases in the case of the F-type in-medium four-quark condensate while the energy slightly decreases when we use the QM-type in-medium four-quark condensate. The energy shift of  $\Lambda$  in nuclear matter has been extracted from the binding energies of hypernuclei (see Ref. [81] for a review), which is consistent with our result of the QM-type density dependence. This consistency supports that the QM-type density dependence is more realistic than the F-type, namely the density dependence of the four-quark condensate of Eq. (39) is quite weak. As for the  $\Lambda_b$  analyses with the QM-type density dependence, the energy is almost independent of the density, which implies the difficulty of forming a bound state. Comparing the results of  $\Lambda_b$  with those of  $\Lambda_c$ , we find that the values of the energy shift, the effective mass shift and the vector self-energy of  $\Lambda_b$

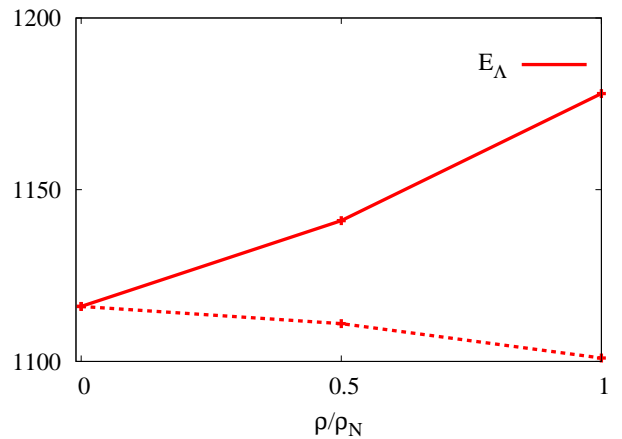


FIG. 5: The density dependences of the energy  $E_{\Lambda}$  [MeV]. The solid line shows the result with the density dependence of the four-quark condensate according to the factorization hypothesis while the dashed line is the case where its density dependence is estimated from the perturbative chiral quark model.

and those of  $\Lambda_c$  are of the same scale. These behaviors come from the small in-medium modifications of the four-quark condensate and  $\langle q^{\dagger}q \rangle$ .

## V. SUMMARY AND CONCLUSION

We have studied the properties of  $\Lambda_c$  in nuclear matter by using the QCD sum rule. To improve the  $\Lambda_c$  QCD sum rule, we have taken into account the first order  $\alpha_s$  correction and the higher order condensate terms in the OPE and then construct the parity projected QCD sum rule. The employed  $\Lambda_c$  interpolating operator is the scalar diquark-heavy quark type whose structure is same as the quark model picture of the  $\Lambda_c$  ground state. In the OPE side, the four-quark condensate is dominant in vacuum, as  $\langle \bar{q}q \rangle$  does not appear due to the structure of the interpolating operator. Therefore,  $\Lambda_c$  feels the in-medium effects mainly through the four-quark condensate. We find that our interpolating operator strongly couples to the positive parity state while the coupling to the negative parity state is weak.

From the analysis of  $\Lambda_c$ , the density dependences of the energy  $E_{\Lambda_c}$ , the effective mass  $m_{\Lambda_c}^*$  and the vector self-energy  $\Sigma_{\Lambda_c}^v$  are obtained. We have found that the results depend strongly on the density dependence of the four-quark condensate. We have considered two cases. The first one is the density dependence according to the factorization hypothesis (F-type) and the second one is estimated from perturbative chiral quark model (QM-type). The density dependence of the QM-type is much weaker than that of the F-type. The result with the F-type in-medium four-quark condensate shows that the

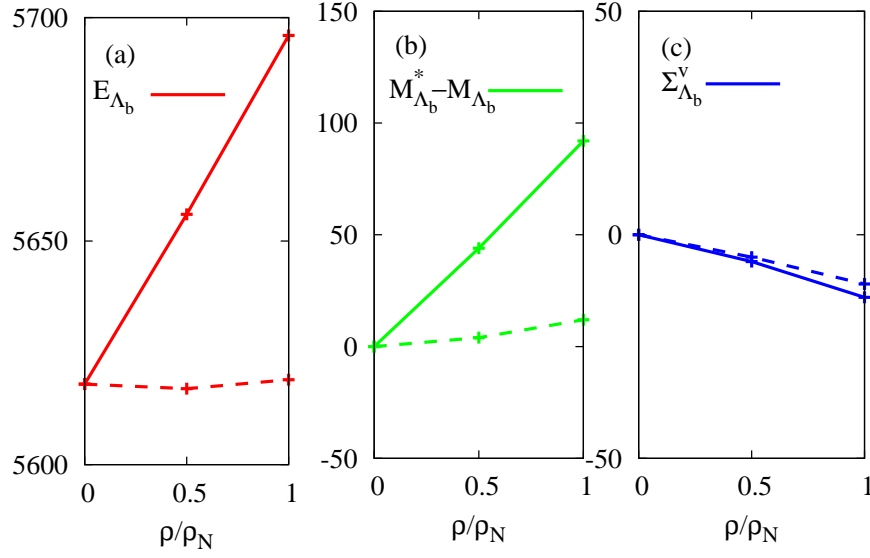


FIG. 6: The density dependences of (a) the energy  $E_{\Lambda_b}$  [MeV], (b) the effective mass  $M_{\Lambda_b}^*$  [MeV] and (c) the vector self-energy  $\Sigma_{\Lambda_b}^v$  [MeV]. The solid lines show the results with the density dependence of the four-quark condensate according to the factorization hypothesis while dashed lines is the case where its density dependence are estimated from the chiral quark model.

energy and the effective mass increase while the vector self energy decreases in nuclear matter. On the other hand, the result using the QM-type is that the energy, the effective mass and the vector self-energy decrease in nuclear matter, which indicates the  $\Lambda_c$  bound states in nuclear matter. The obtained binding energy is about 20 MeV at the normal nuclear matter density. The sensitivity to the in-medium modification of four-quark condensate implies that  $\Lambda_c$  is useful as a probe of the density dependence of four-quark condensate. Comparing the values of  $m_{\Lambda_c}^*$  and  $\Sigma_{\Lambda_c}^v$  with those of  $\Lambda$  in the previous QCD sum rule analyses, we find that their values are considerably different from each other. This implies the large violation of the naive expectation where exchanged light mesons only coupled to light quarks in the baryon. From the viewpoint of the OPE expression, the discrepancy can be understood as follows. The smallness of the vector self-energy come from the  $\langle q^\dagger q \rangle$  term whose contribution plays a dominant role to determine the vector self-energy in the case of the nucleon and  $\Lambda$ . The Wilson coefficient of this term becomes small as  $q_0$  goes close to  $m_Q$ . Due to this property, when we investigate  $\Lambda_Q$  whose energy is close to the heavy quark mass, the contribution of  $\langle q^\dagger q \rangle$  to the vector self-energy is suppressed. As a result, the value of  $\Sigma_{\Lambda_Q}^v$  becomes small. For the effective mass, its value is also small as there is no contribution from the quark condensate  $\langle \bar{q}q \rangle$ .

We have applied the parity projected QCD sum rule

to the analyses of  $\Lambda$  and  $\Lambda_b$  in nuclear matter. It is found that the in-medium modification of the energy of  $\Lambda$  depends on the density dependence of the four-quark condensates and the result with the QM-type density dependence is qualitatively consistent with the experimental results. This consistency supports that the density dependence of the four-quark condensate is quite weak. The results of the  $\Lambda_b$  analyses using the QM-type density dependence show that the energy is almost density independent, which implies the difficulty of forming bound states. Comparing the results of  $\Lambda_b$  with those of  $\Lambda_c$ , we find that the values of the energy, the effective mass and the vector self-energy of  $\Lambda_c$  and those of  $\Lambda_b$  are of the same scale. These behaviors come from the small in-medium modification of the four-quark condensate and  $\langle q^\dagger q \rangle$ .

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